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ABSTRACT

Difficulty in comprehending mathematical concepts is a consequence of the system of knowledge acquisition prevailing in schools, and more specifically, of the typical pattern of classroom interaction. In the common task-setting/task-solving pattern, the teacher divides the subject matter into segments. Students are involved interactively in developing isolated solution proposals to the tasks set for them. However, the subject matter is fragmented and decontextualized. This approach is particularly inadequate for mathematics instruction. An example of student-teacher classroom interaction illustrates how focusing on the teacher's agenda deprives the student of the opportunity to solve the problem in his or her own way. Placement of the problem in its appropriate context is necessary for thorough comprehension of mathematical concepts. (MSE)

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Rainer von Kügelgen

INTERACTIONAL PROBLEM-SOLVING IN MATHEMATIC-LESSONS

-AN TRANSCRIPT-ANALYSIS-

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I am going to compare two *lingual patterns of actions*: Task-setting/task-solving (t)[1] and instructional problem-solving (p)[2]. Lingual patterns of actions according to Ehlich & Rehbein 1986 are defined as special forms of handling purposes. They are composed of mental actions and interactions.

Communication in institutions like school, is largely following lingual patterns of actions (Ehlich/Rehbein, 1979, Rehbein, 1977). Patterns are social structures of depth. Its sequencial positions are not mechanically formed out on the surface of speech. They can only be found by analyzing the specific realization of action-purposes of the co-actants. Transcript-analysis is the base of pattern-analysis.

One of the most frequently applied patterns in school is *task-setting/task-solving* (Ehlich/Rehbein, 1985). This pattern aims at the organization of accelerated acquisition of knowledge in comparison to out-school learning. Characteristic for this pattern is that the teacher disposes of the propositional contents of the given problem and of the steps, necessary to come to the solution. The interactional achieving of the solution thus is pre-planned by repeatedly going through the task-setting/task-solving-pattern. From the teacher's view the complete problem is reconstructed step by step in the successive task-solutions.

From the students' view the result is nothing but an addition of tasks and solutions in temporal succession. To find out the function and meaning of each separate solution they would have to reconstruct its specific localisation in the context of the entire problem. Without this *contextualisation* of the acquired parts of knowledge the solutions stay worthless for the on-going of the problem-solving process and any other creative application.

Particular problem-solving by a student requires on the teacher's side leaving his own propositional plan and developing receptivity for the students'. Normally the teacher integrates the students' solution proposals by taking them as catchwords to realize the next-following step in his plan. Hence the task-setting/task-solving-pattern includes the danger of a *paradoxical contradiction*: In order to organize accelerated achievement of knowledge it removes the contextualisations and the reconstruction of the problem's entire propositional plan from the action side of the students thereby destroying the presuppositions of learning.

On the students' side the results of task-solving are *isolated dictionary-like elements of knowledge* and unintellegible *operative schemes* about how to use them. Wher-

[1] v. the diagram in the appendix, p. 8

[2] v. the diagram in the appendix, p. 9

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ever a student acquires comprehensive knowledge he must do this against the de-contextualizing pressure of the task-setting/task-solving-pattern. This kind of teaching-learning-discourse has deep traditional roots. A famous example in mathematics is Plato's dialogue between Socrates and Meno's slave concerning the duplication of a square. (Struve/Voigt, 1988).

The lingual action-pattern of instructional problem solving, found out by transcript-analysis, avoids the described dangers by the necessity for the students to run through its position of *integration of knowledge element in context* (p 1) of the whole problem and to develop from here his further *solution proposals*(p 5/6). Correspondingly to this the teacher must run through the position of mental *reception* (p 7) and develop from here his further steps, as for example *inquiry* (p 12) in case of *incomprehension* (p 11) or *deficit analysis* (p 16) respectively *deficit supposition* (p 19) and *support* (p 20).

An essential purpose of interaction in school is the mediation of knowledge. An important part of knowledge represents further experience or solvings of social problems. It is not possible to re-discover or to repeat all experiences that are condensed in the knowledge that has to be learnt in school. Therefore it is necessary to find possibilities for an accelerated achievement of knowledge.

The task-setting/task-solving-pattern is the natural -that is to say the spontaneous or unconscious- way to manage this in school. It works in the following way: A complex problem (as for example cosine-theorem) is distributed into parts according to the teachers plan (t 1). These parts are given separatedly and successively as tasks to the students (t 2). The partition goes on continously until the fragments become dissoluble for the students. I found for example about fifty different tasks within 25 minutes of a lesson. The teacher lets the students, if necessary, run several times through the task-setting/task-solving-pattern to find the solution (t 15). Doing this, he is feeding in the task in kind of hints (t 14) more and more of the propositional contents of the problem. It appears that the students in this way can solve step by step the complex problem. And in fact they may produce all the single elements of knowledge (t 7, 17,18) that the problem is composed of.

But nevertheless the function of the fragments of knowledge for the entire problem stays unclear for the students, because their context is not reconstructed. In the students mind there is only an addition, but no totality, no entirety. Therefore it is normally only the teacher who notices the arrival of the solution. For the students the fragments of knowledge are connected just by their succession in time not by their *function* in the frame of the whole problem. The synthesis of the fragments stays the job of the teacher, because the synthesis reclaims the integration of the elements of knowledge in the context of the problem. This work of integrating in the context is the essential thing for the achievement of knowledge. Whithout having done it, knowledge stays worthless and uncapable for creative application.

In the following I want to discuss an example of problem-solving in school. In this example are realized essential positions of this alternative pattern of achievement of knowledge. It is drawn out of a transcript of a mathematic-lesson concerning cosine-

theorem[3]. The students are in a special foreigner class of the tenth grade. They live at this time about three years in Germany and are about seventeen to twenty years old. The small numbers count the segments of the discourse. 'T' stands for teacher, 'Bi' for Birol and so on.

First of all the teacher modifies the categories following to that he had in advance criticized Birols behaviour during the last lesson:

(1 Okay, hmmm, now **Birol** ((-*1sec*-)) was considering, earlier, back at the very beginning of the hour, how we can find out h-ce, and **I** said at that point, "Nah, just leave it!", and then he got mad and ...4 **slept**. 5 **Sure**, if you do not want to know, what someone says...)

Doing so the teacher in a certain way apologizes for his oppression of Birols idea. The social rehabilitation precedes the professional rehabilitation and renders it possible. Then the teacher renounces of his plan as the organizing principle of the interaction and announces Birol's instead of it:

(6 Okay. And we wanna listen to that now. 7 Now, this is... **Bi´rols va`riant!**).

The planning competence (t 1) thus is given from the teacher to the student (p 1). This is registrated by the students as an exceptionell action:

(15 Our professor).

And in fact the readiness of the teacher to leave his plan and to accept the student ones (t 1 –> p 1) is a prior condition for realizing problem-solving in school.

Now Birol starts with his idea. In order to recapitulate his idea and to fix again his aim he mentally goes back to the situation where this idea was born

(17 Where was that thing anyway?... 19 It was at the beginning where wejaaa here, in this triangle ((-*indicates DBC*-)) where we wanted to find out h-ce.).

This situation represents for him the constellation of the problem, the difficulty or so to say the resistance of the subject matter against understanding. It represents also the interest of action to overcome the problem.

(19a And since we of course did not know... 21 ((--*2sec*--)) 22 Yeah...)

By these utterances the context of the problem's development until the actual phase is reconstructed.

This is the base for the following summary of the way to the solution that Birol intends to go. Birol gives this summary and makes his plan clear:

(22 Yeah... 23 If we, hmmm, **come in** this triangle ((-*indicates ADC*-)) 24 We have of course, hmmm applied the Pythagorean Theorem here. 25 At this point I would like to, hmmm, hmmm, apply **sine** rule. 27 In/In this, hmmm triangle ((-*indicates ADC*-)))

The reconstruction of the context and the construction of his plan is represented in (p 1) of the pattern problem-solving in school. To find the next steps towards the solution (p 5) Birol now divides the problem in its single parts under the categories of known and unknown:

(28 And that...we know...this **line** ((-*indicates b*-)), 29 and we know this ((-*indicates alpha*-)) angle ((--*2sec*--)) 31 We know this angle too ((-*indicates angle ACD*-)). 35 We know this angle, **everyone**. 37 Exactly like here, umm I do not know. 40 That is, ehm, ninet/.../hundred

[3] v. appendix, p. 7

eighty... minus ninety minus alpha...44 That is gamma.)

From here he finds the concrete steps towards the solution by putting the determined parts in the sine-theorem-equation:

(67 ((----------*writes:* $b / 1 = h_C / \sin \alpha = AD / \sin 90^o - \alpha$ ----------)). 68 W' **know** be. 69 ((*-writes-*)) 73 And then... ninety...there... 74 W' know that too, that is one, ...equals. 77 We also know what **alpha** is, 78 but we do not **know** h-ce. ((*-writes 5sec-*)). 80 Sine alpha... and then... 81 We know gamma 88 Oh, yeah, right, A-Ce-De. 91 A-De..., line A-De equals. 92 We know line A-De. ((*-writes 2sec-*)) 93 No, we do not know ((*-erases-*)). 95 We dont know it. ((*-writes 3sec-*)) 96 A: nd we know...yeah ss gamma, so it seems. 103 Sine angle A-Ce-De.)

It would now be possible to determine both unknown magnitudes, -straight line h_C and straight line AD-, out of this given equation. And Birol makes clear, that he wants to do so when he says:

(137 Well, **for the first time** we wanted to find out h-ce ...)

But the teacher does not grasp what Birols idea consists on:

(26 The **sine**-rule?).

He thinks Birols idea is nothing but another way to determine only the unknown magnitude h_C as he uttered above:

(1 Okay, hmmm, now **Birol** ((*-1sec-*)) was considering, earlier, back at the very beginning of the hour, **how we can find out h-ce**...)

The teacher is mentally not leaving the frame of his plan (t 1) and obviously subordinates Birols idea to it. According to the teachers plan h_C has to be put in a Pythagorean-Theorem-application in order to continue. Therefore he is going to de-focus, to de-topicalize sine-theorem as an useless complication for the on-going of the problem. During the next two hundred thirty seven segments (s106-342) the teacher takes back the interactional initiative and follows ideas of his own as for example an excourse concerning "what is b divided by one?"

This interruption and de-topicalization destroys the development of Birols idea. Thereafter he can merely *remember the results* of his idea, as shown among others by his past-tense-application:

(342 Yeah. 343 But what I really **wanted** to do...345 That wasn't so interesting. 346 Then, I **could**, ehm, also figure out A-De.)

This shows that his context is destroyed:

(349 Yeah, what do we really know?)

Finally he determines h_C with the help of the teacher:

(366 Whether I can figure it out...? 370 That is be times sine alpha. 372 ((*-writes 3sec-*))...sine alpha. 374 We know now h-ce.)

But now he is not capable to put h_C in the right equation. He has lost the origins of his own idea as given before at the beginning of his problem-solving-process when he said:

(19 It was at the beginning where wejaaa here, in this triangle ((*-indicates DBC-*)) where we wanted to find out h-ce.)

and he describes his confusion:

(383 I have to put it **somewhere**. 387 ((-----*5sec*-----)))

This incapacity of putting an element of knowledge in its context in the frame of the entire problem (p 1) is the typical result of an interaction that follows the task-

setting/task-solving pattern. It shows that the interaction fell back from problem-solving to the task-setting/task-solving-pattern. In this pattern the student is just an assistant for the teacher to do the unskilled work of task-solving. Task-solvings are delivered as fragments to the teacher without any conscience about their function. Their further development stays in the sphere of responsibility of the teacher.

In consequence of the destruction of his context Birol falls back into the standard-version of cosine-theorem-demonstration:

(388 ((--*sighs*--)) We can... 389 Where did we put that? 390 h-ce... ((-*searchs for 3sec*-)) 391 Well... be-square...here! ((-*Birol wants to apply* $h_c = b \cdot \sin \alpha$ ***into the Pythagorean Theorem for the triangle ADC:*** $h_c^2 = b^2 - (AD)^2$.-)))

With the help of the teacher he recapitulates later on (s392-593) his way of solving until the given point and finds the correct equation to put h_c in. Regulated by this he succeeds in reconstructing his context-consciousness and continues to determine straight-line AD out of the above given sine-theorem application (v. s25-103, especially s67) that is central for his problem-solving:

(595 Line A-De....600 Ummm... 601 be ti/ 601a be times line.../ ((-***indicates the sine-rule, s67***-)) 602 Well, I guess I really ought to write that down.)

The other students remember at that point the standard-demonstration of cosine-theorem according to which straight-line AD has to be determined with a cosine-application of angle alpha:

(Yi 598 Now we have to calculate the cosine, yes. Mu 599 Here's the thing with the cosine, right.)

Only now the teacher recognizes the difference of Birols idea to the standard-demonstration as an alternative, autonomous demonstration:

(603 What you are doing there is **really** interesting, indeed.)

Only now the teacher is reconstructing the mental processes that Birol made in (p 1) of the pattern problem-solving in school. This reconstruction (p 7), is a mental reception. Reception according to (p 7) is the corresponding position in the action-sphere of the teacher to (p 1) in the action-sphere of the student. The act of reception according to (p 7) of the pattern problem-solving in school must not be mistaken for (t 8, 9) of the task.-setting/task-solving-pattern. (t 8, 9) is a decision point where the teacher is merely deciding to overtake (t 16) or to refuse (t 10) those parts out of the students solution proposal (t 7) that fits to accomplish his plan.

The reception according (p 7) cannot be realized as a fragment but reconstructs the entire mental process of the student. The reconstruction or understanding of the entire utterance and idea of the student leads to a different quality of interaction in (p 12, 15, 20, 22) of the problem-solving-pattern compared to the positive or negative assessment (t 11, 17) of the task-setting/task-solving-pattern. This different quality is well-noticed by the students:

(Be 605 You look so **pleased**....)

but also by the teacher who is astonished at the dimension of Birols idea:

(606 ((-*laughs*-)) 607 Yes, I'm pleased. 608 ((-*laughs*-)) 610 I have **never** seen **anything** like

that before...!)

When Birol has completed the equation, another student recognizes the determination of straight-line AD that means of the problem itself:

(Yi 625 Now nothing is left to find out.)

This student has put Birols supposition in the context of the problem (p 1) and so found out its function as solution of the entire problem. So (p 22), the confirmation of the solution by the teacher is just a kind of *ratification* of knowledge that was found by the students themselves. It has a completely different quality than the positive assessment (t 17) of the task-setting/task-solving-pattern. (t 17) is for the students the *creation* of knowledge that is to say normally of facts.

The utterance (625 Now nothing is left to find out.) is a further proof that the interaction follows the problem-solving-pattern: In the task.-setting/task-solving-pattern nobody but the teacher knows when the solution is reached because only he knows the function of the determined element of knowledge in the context of the whole problem.

This briefly interpreted example shows that the structures of the task-setting/task-solving-pattern make their way even under favourable conditions when the teacher intends willingly to give the planning-initiative to a student and self-criticizes his oppression of a students idea (v. s1-7). To avoid the enforcement of these natural structures of institutional interaction, consciousness is reclaimed concerning necessary positions of problem-solving that means essentially the contextualization by the students (p 1) and the reception (p 7) by the teacher.

A further important aspect of pattern analysis of interaction in school can only be mentioned briefly[4]: The elements of knowledge in the task.-setting/task-solving-pattern are separated from the context in which they make sense. Therefore the students cannot determine the demanded knowledge according to the logical and professional contents of the subject matter. And also they cannot determine the demanded knowledge according to the specific methods of the subject matter. Instead of that they must develop other methods in substitution to produce the demanded elements of knowledge. The students find these ersatz-methods in interpreting interactional and communicative evidences and in functionalizing their pattern-knowledge with an amount of techniques, tactics and strategies. Pattern analysis can analyze these substitutional methods.

Problem-solving in school needs concerning knowledge context-reconstruction and reception and additional to that concerning methods criticism and consciousness relative to both, substitutional and professional methods.

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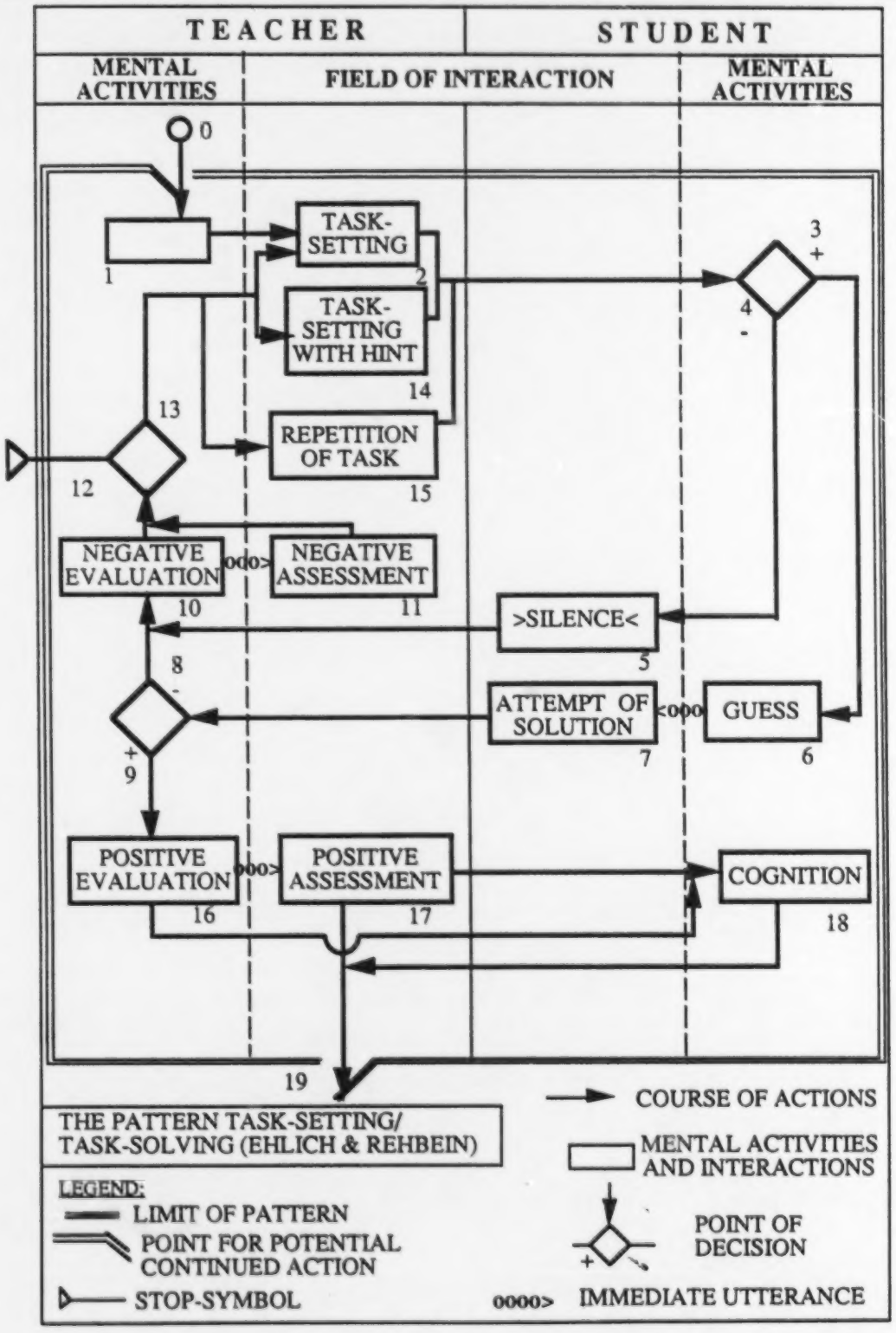
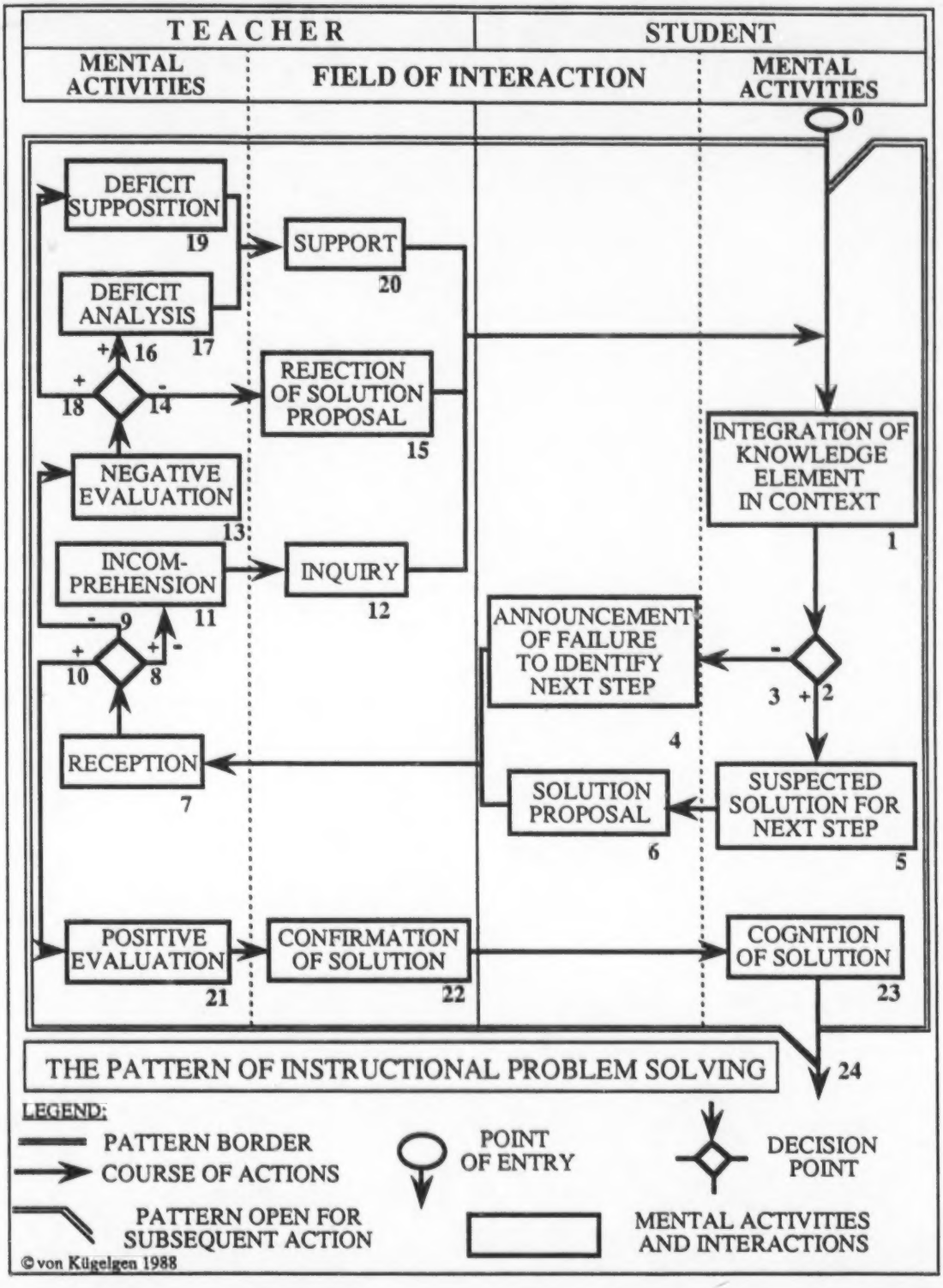
[4] cf. v. Kügelgen, 1990: Comprehension and communicative evidence in operative mathematics instruction

APPENDIX

<<

T 607 Yes, I'm pleased. 608 ((-*laughs*)) 610 I have **never** seen **anything** like that before! ((*laughs*))
Bi ((-*writes*-)) … is equal to… 620 Sine …angle A-…Ce-…De. 624 And now/
Yi 625 Now nothing is left to find out.





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